A Tribute to the Memory of

Sonya Kovalevsky

(1850–1891)

by fil dr Ingegerd Palmér

Royal Swedish Academy of Engineering Sciences (IVA)

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Presented at the 2011 Annual Meeting
of the Royal Swedish Academy of Engineering Sciences

by fil dr Ingegerd Palmér

The Royal Swedish Academy of Engineering Sciences (IVA) is an independent, learned society that promotes the engineering and economic sciences and the development of industry for the benefit of Swedish society. In cooperation with the business and academic communities, the Academy initiates and proposes measures designed to strengthen Sweden's industrial skills base and competitiveness.

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Foreword

Every year the Royal Academy of Engineering Sciences (IVA) produces a booklet commemorating a person active in Sweden whose scientific, engineering, economic or industrial achievements were of significant benefit to the society of his or her day. The person to be recognised by the booklet must have been dead for at least 100 years. The Commemorative Booklet is published in conjunction with the Academy's Annual Meeting.

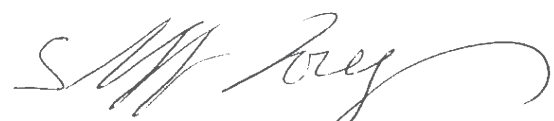
This year Sonya Kovalevsky, the first woman to become a professor of mathematics in Sweden in modern times, is honored in IVA's commemorative booklet. Sonya Kovalevsky was bo rn in Russia but moved abroad to study mathematics. Her interest and determination took her around Europe, via Berlin where she met her lifelong friend and mentor Gösta Mittag-Leffler, to Sweden and the brand new university Stockholms Högskola. This young academic institution was progressive enough to offer high-level academic positions to women. Sonya is still seen as an inspiration around the world, especially among young women interested in mathematics.

In 1888 Sonya won the prestigious Prix Bordin for her mathematics paper *Sur le problème de la rotation d'un corps solide autour d'un point fixe* (On the rotation of a rigid body around a fixed point). It was considered excellent enough to warrant awarding her double the prize money.

She was also an author of fiction. In 1887 she co-wrote the drama *Kampen för lyckan* (The Struggle for Happiness) with Anne Charlotte Leffler, and in 1889 her autobiography *Ur ryska lifvet: Systrarna Rajevski* (From a Russian Life: The Rayevski Sisters) was published.

Sonya Kovalevsky died in 1891 in Stockholm of influenza complicated by pneumonia and is buried at Norra begravningsplatsen, a cemetery in Solna just north of Stockholm.

Björn O. Nilsson
President of the Academy



Staffan Josephson
Chairman of the Medals Committee

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Introduction

Sonya Kovalevskaia was indeed a remarkable person. 120 years after her death she is still recognized for her contributions to mathematical research, but even more as a role model for her strong determination to work within science and her success in fulfilling that ambition. She is one of the very first women in modern times to achieve full professional status and recognition within the natural science community. It is hard for us living in the 21st century to fully understand how very unusual and difficult it was in the late 19th century for a woman to enter academia at all, let alone the natural sciences field, the jewel of them all at the time. She grew up at a time when progressive ideas were developing among young people in Russia. She had the opportunity to learn some mathematics and this made her determined to study at a university. She pursued that path enthusiastically until she eventually received a doctoral degree and go into a mathematical career.

Sonya Kovalevskaia was the first woman to become a professor of mathematics in modern times, and it happened in Sweden at a brand new university, Stockholms högskola, which had opened it doors in 1878. It was a private institution not bound by university laws. It did not initially have the right to award degrees, but enjoyed many freedoms. The university had a diverse student body and was not obliged to

obey the employment rules for state institutions where women were not permitted to hold high positions at the time.

Sonya had a wide variety of interests and talents: mathematics and sciences as well as literature, politics and social development. She was an author of fiction, and the modern media would have revelled in her amorous exploits and personal affairs.

It is appropriate that Sonya Kovalevskaia is being honoured in this way in 2011, 100 years after the first PhD in mathematics was awarded to a Swedish woman. She was Louise Petrén who graduated from Lund University with her thesis entitled *Extension de la méthode de Laplace*. She did not continue with research, but worked for some time as an actuary at an insurance company. It was well over 100 years after Sonya Kovalevskaia's death that Sweden's second female professor of mathematics was appointed. Today, out of around 70 mathematics professors at Swedish universities, the author has found four women.

Sonya Kovalevskaia is still highly respected around the world and seen as in inspiration, especially among women interested in mathematics. There are regular Kovalevskaia High School Days, visiting women's professorships at universities, and various conferences and seminars in her name, most of them focusing on opportunities for women to pursue scientific careers. The Swedish Mathematical Association holds annual Sonya Kovalevsky Days to promote an interest in mathematics among secondary school girls and boys.

Her name

Sonya Kovalevskaia was born Sofia Vasilevna Korvin-Krukovskaia. She married Vladimir Kovalevsky and adopted his surname in female form. Sonya is a diminutive of Sofia and is the name she used as an adult. *In Swedish her name is written as Sonja Kovalevsky.*

Young Sonya

Family and childhood

Sonya Kovaleskaia was born in Moscow in 1850. Her father, Vasily Vasilievich Krukovskoi (1801-1875), served in the army and became a general in 1852. He was quite well educated, spoke English and French fluently and had a fairly good knowledge of mathematics and natural sciences. He was also acquainted with some of the leading intellectuals of his day.

Her mother, Elisaveta Fedorovna (1820-1879), was the daughter of a military man who had made notable scientific contributions in geodesy and geographical maps. Her grandfather was a famous astronomer called *F.I.Schubert*, a member of the Imperial Academy of Sciences who corresponded with Gauss, Laplace and Bessel, among others. Elisaveta was a lively and beautiful woman, an accomplished pianist and amateur actress.

Sonya was their second child. Their first, a daughter Anna Vasilevna, called Aniuta, was born in 1843 and the third was a son, Vasily Vasilievich, born in 1855.

Sonya's first few years were spent in Moscow and in Kaluga, an artillery station south of Moscow. In 1858 the family moved to her father's Palibino estate, halfway between St. Petersburg and Kiev. One of the reasons for the move was the need to oversee the estate amidst rumours of the emancipation of the serfs.

Sonya has described her childhood as unhappy. She admired her elder sister Aniuta who was blonde, attractive, very social and thus fitted well into the ideal at the time for a girl. Sonya was dark and had an intense and serious personality; she felt clumsy and awkward in social contexts. She became her father's favourite, however, and this is believed to be one of the reasons he subsequently allowed her to enter academia and engage in activities not common for young women.

In her autobiography, Sonya Kovalevskaia makes particular

Mature Sonya

mention of two uncles. Her mother's younger brother was her idol. He told her stories, not about dragons and princesses but about biology, infusoria, alg*ae* and how coral reefs are formed. One of her father's brothers had a deep respect for and interest in mathematics and he told Sonya at an early age about squaring the circle, asymptotes and many other mathematical concepts.

Sonya Kovalevskaia also mentions in her autobiography a room at their estate that had walls covered with notes from a calculus course. They fascinated her and she believed this is how she became familiar with symbols and formulas.

A family tutor was hired when Sonya was 9 years old. She quickly learned basic mathematics from him and continued on to algebra and geometry. She begged her father to bring books on mathematics back from his visits to St. Petersburg and Moscow. She was given a physics textbook and encountered trigonometry for the first time, but her tutor was unable to explain it to her. While struggling with the formulas she came to the conclusion that a chord would serve her just as well as the unknown sine. She was then able to work through the book without much difficulty. She eventually met the author who was surprised and excited to realise that Sonya had used the same approach as Ptolemay and others in the early development of trigonometry.

The author was a professor of physics in St. Petersburg. He was liberal-minded and in particular he believed that women were as capable of intellectual achievements as men. He strongly urged his friend, Sonya's father, to allow Sonya to study; as he put it: it would be a crime to let her mathematical abilities go to waste.

During her adolescence, Sonya's sister Aniuta became a serious young lady, turning her back on fancy clothes, balls and courtship. She focused on studying, sent for books from St. Petersburg and taught the servants' children how to read. She also took an interest in the living conditions of the peasants and especially the women. She learned about the nihilist circles in St. Petersburg that attracted young men and women who questioned traditional tsarist Russia. She had great faith in the power of education and natural sciences, believed strongly in the equality of women and wanted to find a way to support the common people.

Aniuta begged her father to allow her go away to study. He ultimately denied her this in an angry outburst and their relationship deteriorated.

Aniuta wrote stories and in 1864 she sent two of them to the journal edited by the Dostoevsky brothers. Her novels were accepted and to her surprise, she was paid for them. Dostoevsky took an interest in Aniuta and when they met in Petersburg he eventually asked her to marry him. She refused. Sonya also met Dostoevsky and, at the age of 15, became infatuated with him and jealous of her sister.

Sonya was very impressed that her sister now was a writer. She herself wrote poems from time to time and greatly respected anyone whose words appeared in print.

Sonya also developed an interest in politics, influenced by her sister and others, and for a while dreamed of participating in the Polish rebellion. Her main political views seem to have remained constant throughout her life.

In search of education

In the end Sonya's conservative father recognized his daughters' talents and determination to study, and in 1867 Sonya and Aniuta were allowed to go to St. Petersburg. Sonya was to study mathematics under Professor Strannoliubsky, a well-known pedagogue and a champion of the right of women and workers to have an education. A biographer of Sonya writes: *"Sofia was in ecstasy over Strannoliubsky. He not only taught her mathematics, he undoubtedly inspired her with enlightening aspiration."* (Koblitz p. 61)

It was still, however, impossible for a woman to receive a formal university education in Russia. For a while women were admitted semi-officially, but in 1862 all institutions of higher education were closed to all but formally enrolled students. Sonya's studies with Professor Strannoliubsky made her even more determined to pursue a career in mathematics. Neither Sonya nor Aniuta was remotely interested in leading the life that upper class women were expected to lead.

They knew that women were admitted as students at some universities in the West. But how could they get there? A woman needed permission from her father or husband to travel abroad and women travelling alone were met with suspicion. An extreme solution to this problem was to find a fictitious husband who would put his "wife" on his passport and allow her to pursue her own interests. If one of a group of

women got "married," the "couple" could act as chaperones and take the entire group to the West.

So Sonya, Aniuta and two of their friends set out to find a fictitious husband. After a number of attempts, a candidate emerged, Vladimir Kovalevsky. He was a publisher with liberal ideas whose works included numerous scientific books. He had refused an earlier proposal for such a marriage, but after meeting Sonya he agreed to "marry" her. Sonya had to resort to quite drastic measures – the threat of a scandal for example – to obtain her father's approval for the marriage which took place in September 1868.

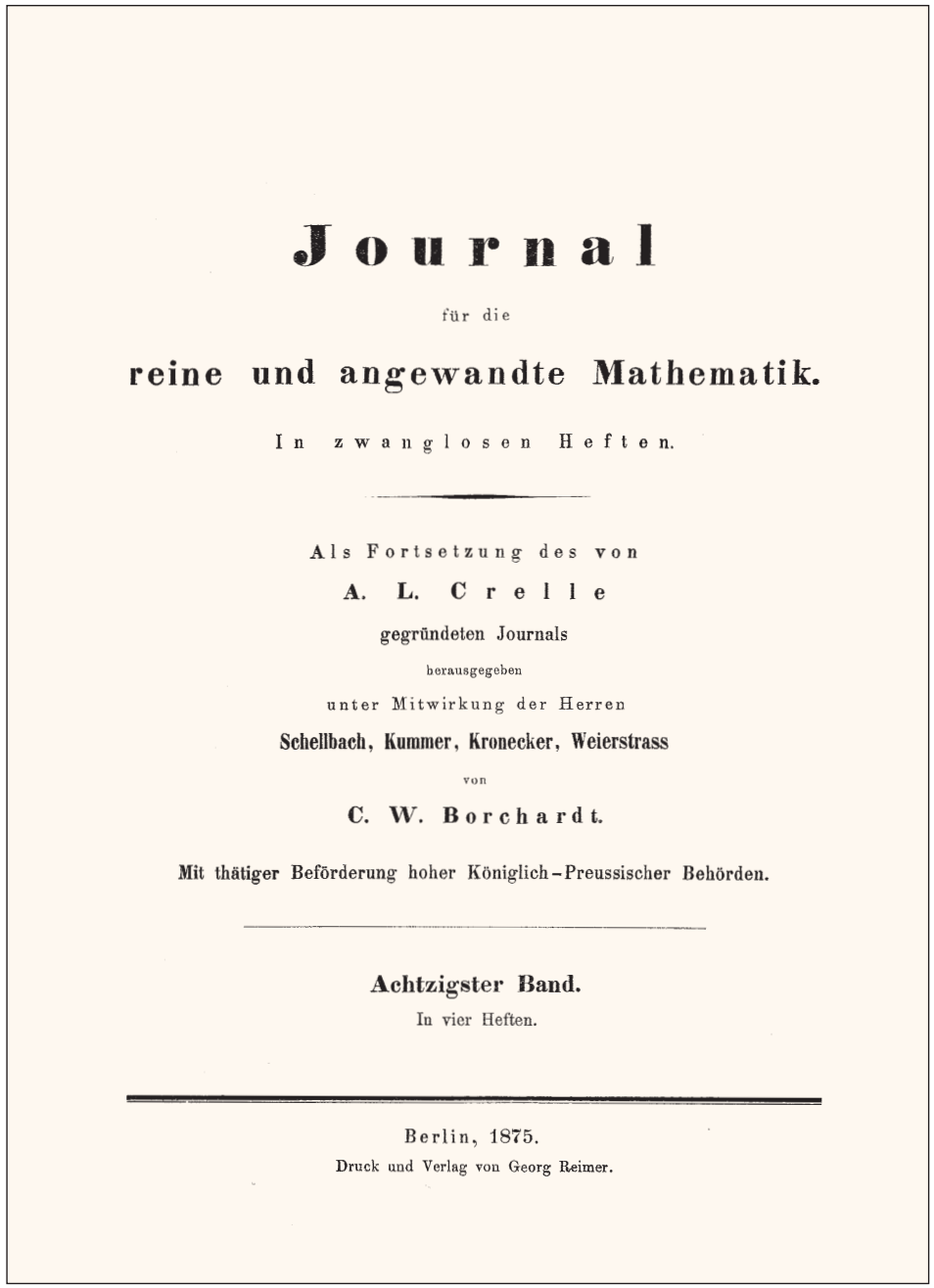
In early 1869 Sonya, Vladimir and Aniuta left Russia. Aniuta went to Paris on her own to join in political activities there. She had to send all of her mail to her parents through Sonya. Sonya and Vladimir went to Heidelberg, and after considerable efforts, Sonya was allowed to attend courses if the professors would grant her permission. She studied physics under Bunsen, Kirchhoff and Helmholtz, and mathematics under a couple of other professors. Considering that her mathematical background was rather weak in comparison to her fellow students, her studies must have been exhausting. Vladimir eventually started work on a dissertation in geology. Although their studies went well, their lives in general were difficult, plagued by financial problems and disagreements about what their marriage actually meant. Sonya wrote a poem entitled "The Husband's Complaint" on how she was making her husband's life miserable by forcing him to keep his promise to allow her to pursue her ambitions.

Becoming a mathematician

After a year in Heidelberg, Sonya took a bold step and went to Berlin to meet one of the mathematics giants of the time, Professor Karl Weierstrass. She hoped to be admitted as a student at the University of Berlin through him.

But university regulations prevented Weierstrass from allowing Sonya to attend his lectures. He did, however, give her a list of problems to solve as a sort of entrance exam. He was impressed by the results. He later told Sonya's friend Anne-Charlotte Leffler: *"She had the gift of intuitive genius to a degree I have seldom found even in my older and more developed students."*. (Koblitz, p. 99-100). Weierstrass offered to give Sonya private lessons twice a week covering the material he was teaching his regular students. He probably never contemplated the idea of Sonya being awarded a degree. What use could that be to a married woman?

They were indeed a strange couple, Karl Weierstrass and Sonya Kovaleskaia. He was 55 when they met; sickly, conservative and leading a very quiet life with his two sisters. He had worked as a teacher in mathematics at a *Gymnasium*. While working as a teacher he also conducted advanced research. In 1854 he published a sensational paper on Abelian functions for which he was awarded an honorary doctorate at Königsberg University. In 1857 he was invited to take up a professorship at the Uni-



Journal

für die

reine und angewandte Mathematik.

In zwanglosen Heften.

Als Fortsetzung des von

A. L. Crelle

gegründeten Journals

herausgegeben

unter Mitwirkung der Herren

Schellbach, Kummer, Kronecker, Weierstrass

von

C. W. Borchardt.

Mit thätiger Beförderung hoher Königlich-Preussischer Behörden.

Achtzigster Band.

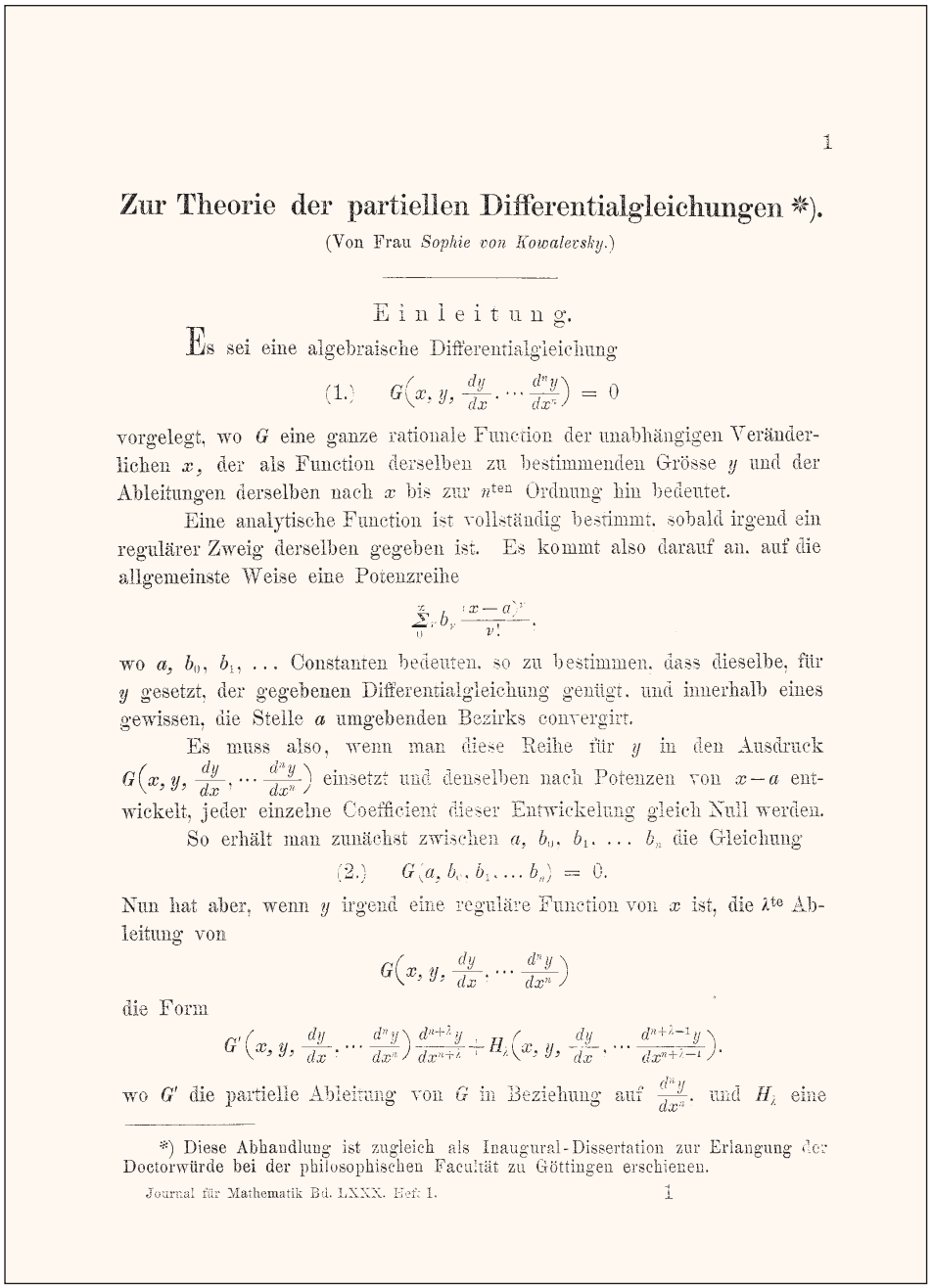
In vier Heften.

Berlin, 1875.

Druck und Verlag von Georg Reimer.

Sonya's first published paper

versity of Berlin. Under his guidance, Berlin became Germany's leading centre of mathematics and he is considered the third most important 19th century mathematician in Germany, after Gauss and Riemann. Weierstrass believed that clarity and stringency are the basis for mathematics. He objected to the more intuitive reasoning of mathematicians such as Riemann. He contributed considerably to the creation of a solid basis for mathematical analysis with his theory on the real numbers and his studies of convergence criteria and proof of existence theorems in the calculus of variations. He also provided a solid foundation for the complex



1

Zur Theorie der partiellen Differentialgleichungen *).

(Von Frau *Sophie von Kowalevsky*.)

Einleitung.

Es sei eine algebraische Differentialgleichung

$$(1.) \qquad G\left(x, y, \frac{dy}{dx}, \cdots \frac{d^n y}{dx^n}\right) = 0$$

vorgelegt, wo G eine ganze rationale Function der unabhängigen Veränderlichen x, der als Function derselben zu bestimmenden Grösse y und der Ableitungen derselben nach x bis zur n^{ten} Ordnung hin bedeutet.

Eine analytische Function ist vollständig bestimmt, sobald irgend ein regulärer Zweig derselben gegeben ist. Es kommt also darauf an, auf die allgemeinste Weise eine Potenzreihe

$$\sum_0^\infty b_\nu \frac{(x-a)^\nu}{\nu!},$$

wo a, b_0, b_1, ... Constanten bedeuten, so zu bestimmen, dass dieselbe, für y gesetzt, der gegebenen Differentialgleichung genügt, und innerhalb eines gewissen, die Stelle a umgebenden Bezirks convergirt.

Es muss also, wenn man diese Reihe für y in den Ausdruck $G\left(x, y, \frac{dy}{dx}, \cdots \frac{d^n y}{dx^n}\right)$ einsetzt und denselben nach Potenzen von $x-a$ entwickelt, jeder einzelne Coefficient dieser Entwickelung gleich Null werden.

So erhält man zunächst zwischen a, b_0, b_1, ... b_n die Gleichung

$$(2.) \qquad G(a, b_0, b_1, \ldots b_n) = 0.$$

Nun hat aber, wenn y irgend eine reguläre Function von x ist, die λ^{te} Ableitung von

$$G\left(x, y, \frac{dy}{dx}, \cdots \frac{d^n y}{dx^n}\right)$$

die Form

$$G'\left(x, y, \frac{dy}{dx}, \cdots \frac{d^n y}{dx^n}\right)\frac{d^{n+\lambda} y}{dx^{n+\lambda}} + H_\lambda\left(x, y, \frac{dy}{dx}, \cdots \frac{d^{n+\lambda-1} y}{dx^{n+\lambda-1}}\right),$$

wo G' die partielle Ableitung von G in Beziehung auf $\frac{d^n y}{dx^n}$, und H_λ eine

*) Diese Abhandlung ist zugleich als Inaugural-Dissertation zur Erlangung der Doctorwürde bei der philosophischen Facultät zu Göttingen erschienen.

analysis through his studies of the power series. Sonya moved to Berlin and Vladimir to moved Jena for his studies. Sonya spent most of her time studying. She spent an occasional evening with Weierstrass and his sisters and received a few visits from Vladimir. This quiet life abruptly changed in the spring of 1871 when Sonya learned that her sister and her husband were trapped in Paris as members of the French commune. Sonya and Vladimir managed to get to Paris and lived and worked in the commune for a month, Sonya as a nurse. When they arrived back in Berlin they received news that the commune had fallen and Aniuta and her hus-

band were in great danger. Aniuta managed to escape to London and received help in finding temporary lodgings from Karl Marx. Through their father's connections, Aniuta's husband was also able to leave France.

For a period of 18 months Sonya wrote three dissertations under Weierstrass' supervision. He considered two of the dissertations good enough to be accepted at any university, but the third he thought so outstanding that Sonya without a doubt deserved a doctoral degree. In this third dissertation she managed to prove that if a power series is obtained formally from a partial differential equation in which only analytical functions occur, the power series must converge. Some 30 years earlier, Cauchy had conjectured that every differential equation with analytic coefficients has an analytical solution, and he himself had obtained substantial results on ordinary differential equations. Now a Cauchy-Kovalevskaya theorem was established.

Weierstrass chose the University of Göttingen for Sonya to attempt to receive her degree as this institution was more liberal than Berlin and Heidelberg. He also managed to arrange for Sonya to be excused from the customary oral examination. He thought her German was too poor and he believed that, since she had no previous university examination experience, she might become nervous when questioned by a group of senior scholars.

Sonya's dissertation received high praise and she was awarded the doctoral degree summa cum laude (with highest honours). She had reached one of her ambitious goals and subsequently returned to Russia.

Back in Russia

Sonya and Vladimir went back to Russia. Vladimir now had a doctoral degree as well. They set out to live a normal married life in St. Petersburg but found it difficult to support themselves. After a couple of attempts, Vladimir passed a magisterial examination – a requirement for teaching above the elementary level. He was not, however, offered any positions that interested him. Sonya was not permitted to take the examination. The couple turned to speculation in real estate in the hope of accumulating enough money to allow them to live an independent life. Sonya inherited her parents' estate, but unfortunately the investments failed in 1879 and the couple lost the financial foundation they had hoped for.

During their stay in St. Petersburg Sonya and Vladimir led an active social life, spending time with scholars and writers such as Mendeleev, Chebyshev, Turgenev and Dostoevsky. Some fifteen years later in a novel she was writing, Sonya included the passage below, no doubt drawing on own experiences:

The reputation of a learned woman surrounded me with a certain aura; acquaintances all expected something of me; two or three newspapers had managed to trumpet me about; and this, for me, completely new role as an eminent woman, although it embarrassed me somewhat, nevertheless amused me very much at first. (*Nigilitska*, The Nihilist Woman, published in 1892).

In October 1878 Sonya gave birth to a daughter, Sophia Vladimirovna, nicknamed Fufa by her parents. Sonya had resumed her correspondence with Weierstrass who encouraged her to consolidate her mathematical knowledge, especially in analytical mechanics and mathematical physics. After the couple's financial crisis, Sonya decided to focus seriously on mathematics again. Upon the suggestion of Chebyshev she presented a paper at the Sixth Congress of Mathematicians and Physicians in early 1880. In the audience was the Swedish mathematician Gösta Mittag-Leffler, a man who was to become a very important influence on Sonya's future. He was also a student of Karl Weierstrass and Weierstrass had written to him about finding a university position for Sonya. In a letter to Sonya in 1878 Weierstrass wrote:

..Mittag-Leffler has become one of my favourite students; he possesses, besides a

Sonya with daughter Fufa

thorough knowledge, an extraordinary ability to learn and a mind directed to the ideal. I am sure that a contact with him would have a stimulating effect on you. (Cooke p.89).

In 1880 Mittag-Leffler was a professor of mathematics at the University of Helsingfors. He made inquiries for Sonya there, but to no avail. The university did not object to hiring a woman, but believed that a Russian professor might attract revolutionary students. As Finland shared a border with Russia, it was thought that this could cause severe difficulties for the university.

Mittag-Leffler was not easily discouraged. In a letter dated 1 March 1881 he wrote to Sonya:

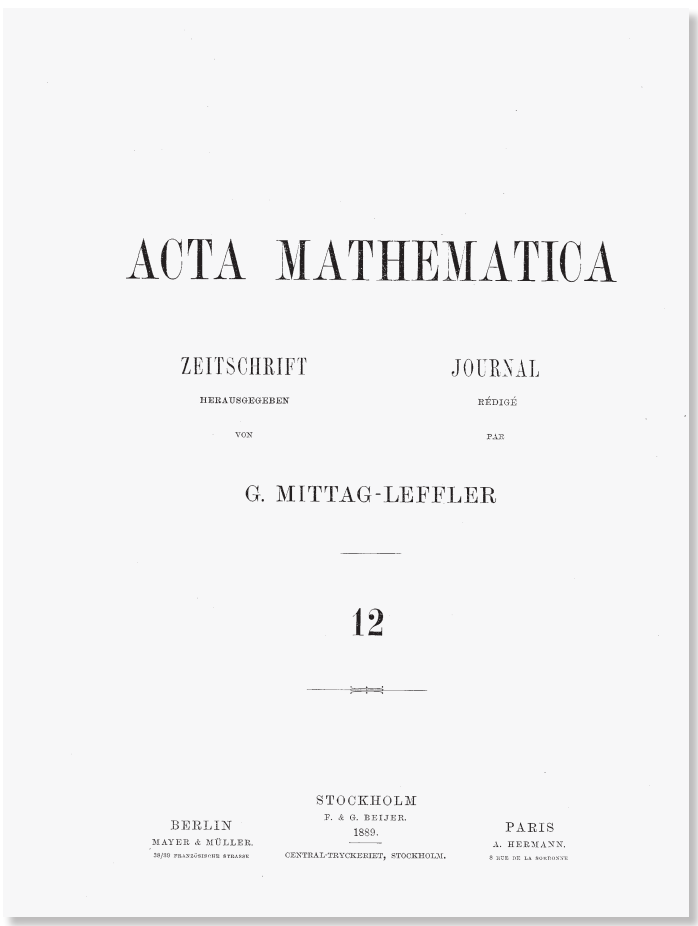
Mme Pokroffsky has told me that you would be willing to accept an official position at any university and consequently I take the liberty of asking if you would authorize me to try to procure such a position for you. I am thinking particularly of the new university which is to be founded in Stockholm. I have been informed "sub secreto" that I will be appointed professor of mathematics there …. and I have no doubt that some day it will be easy for me to arrange an appointment for you, initially as "Privatdozent". The Swedish language will not cause you any great difficulty and the capital of Sweden is one of the most beautiful cities in Europe, where you will also find a very large circle of scholars of great merit. (Cooke p. 90)

Sonya answers thus:

..I never seriously believed in your plans for Helsinki, in spite of wanting very much for them to work out. I also do not intend to place too much hope on Stockholm, although I admit that I would be ecstatic if I were given this opportunity… (Cooke p. 92)

Gösta Mittag-Leffler

Gösta Mittag-Leffler (1846 – 1927) received a doctoral degree from Uppsala University in 1872. In 1876 he accepted a chair at the University of Helsingfors. While there, he published the famous theorem of complex analysis which bears his name.


ACTA MATHEMATICA

ZEITSCHRIFT
HERAUSGEGEBEN
VON

JOURNAL
RÉDIGÉ
PAR

G. MITTAG-LEFFLER

12

STOCKHOLM
F. & G. BEIJER.
1889.

BERLIN
MAYER & MÜLLER.

PARIS
A. HERMANN.

CENTRAL-TRYCKERIET, STOCKHOLM.

In 1881 he joined the newly founded Stockholms högskola. There he began a 30-year career of great mathematical activity. In 1882 he founded *Acta Mathematica* which is still one of the world's leading mathematical journals. He persuaded King Oscar II to honour various distinguished mathematicians around Europe – Hermite, Weierstrass, Poincaré among them – by awarding prizes in competitions. In 1882 Mittag-Leffler married a woman who had inherited a large fortune. He used his wife's money to assemble the best private mathematics library in the world and to build an elegant home in Djursholm, Stockholm.

From there he arranged a wide range of activities to promote mathematics. One important aspect was his extensive correspondence (over 7,000 letters) with practically every notable mathematician in the world. His home was later turned into a national trust site called Institut Mittag-Leffler. It is still a very active and respected centre of mathematical research under the auspices of the Royal Academy of Sciences and is well worth a visit.

Gösta Mittag-Leffler (centre) with family and Sonya

Sonya took up mathematics again. Weierstrass gave her a problem to work on at the beginning of 1881 hoping that she would regain recognition in mathematical circles. The problem involved solving a set of equations describing the displacement of a particle in an elastic medium. Weierstrass had discovered a clever method for constructing general solutions for a large family of linear partial equations (through Green's function for certain initial-value problems) and believed Sonya could use this method.

Sonya started to work on the problem, but quite soon she came up with interesting ideas for solving another problem in mathematical physics which had interested her many years earlier – the Euler equations describing the motion of rotating a rigid body.

She worked on mathematics while bringing up a young daughter and enduring economic uncertainty and a marriage in crisis. Vladimir now held a low-level position at the university in Moscow and was also involved in an oil company. The couple rarely met as both travelled abroad frequently. In spring 1883 Vladimir's professional life collapsed and he committed suicide in April.

Sonya was deeply shocked and starved herself into a coma. She was rescued by a

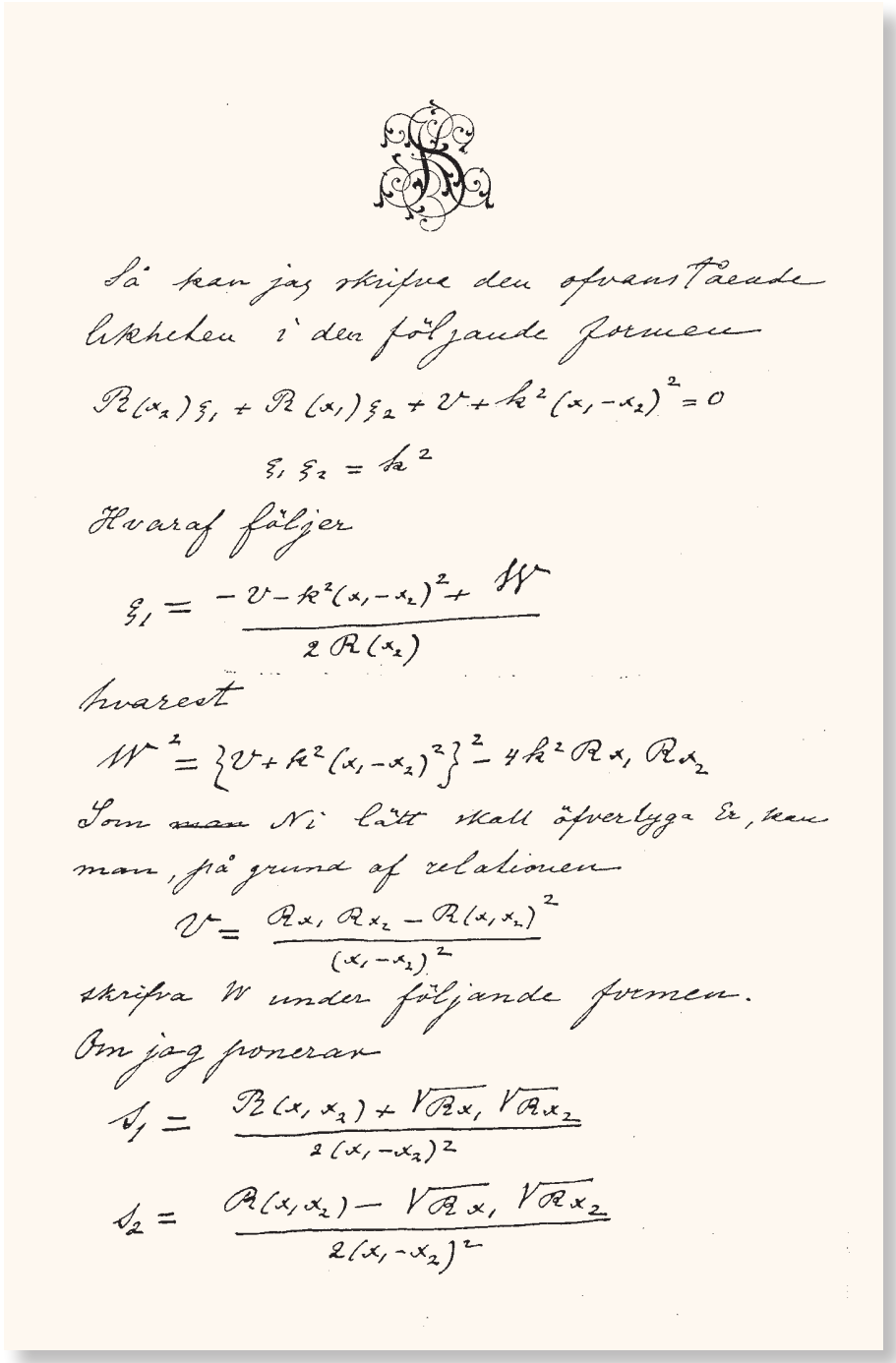
doctor and regained her health, prepared to face her new situation. She needed to finish her research on the problem given her by Weierstrass, but she also wanted to clear up Vladimir's financial mess and free him of various charges. This took her a couple of months.

Now on her own, she desperately needed some kind of financial security. In the summer she was invited to Stockholms högskola as a "Privatdozent." In a letter to Mittag-Leffler she wrote:

I am so grateful to the University of Stockholm, which is the only European university willing to open its doors to me, that I feel disposed already to attach myself to Stockholm and to Sweden as my native land; and I hope that when I come there, it will be to spend many years and to find there a second homeland. (Cooke p. 161)

Later in the letter she discusses what to teach and in what ways she might be able to contribute to Stockholm's intellectual life through all of her contacts with mathematicians and other researchers in Europe. This in particular pleased Mittag-Leffler who was striving to make mathematics an international enterprise with Stockholm as an important hub.

Sonya arrived in Stockholm in November 1883. She left her five year-old daughter in Russia with friends. Her arrival was noted in the progressive newspapers where she was referred to as *"the princess of sciences."* She stayed with the Mittag-Leffler's for some time to get settled in Stockholm and prepare herself for her teaching post. She had to speak German which was her third language. She studied Swedish intensely



Så kan jag skrifva den ofvanstående likheten i den följande formen

$$R(x_2)\xi_1 + R(x_1)\xi_2 + V + k^2(x_1 - x_2)^2 = 0$$

$$\xi_1 \xi_2 = k^2$$

Hvaraf följer

$$\xi_1 = \frac{-V - k^2(x_1 - x_2)^2 + W}{2R(x_2)}$$

hvarest

$$W^2 = \{V + k^2(x_1 - x_2)^2\}^2 - 4k^2 R_{x_1} R_{x_2}$$

Som man nu lätt skall öfverbyga är, kan man, på grund af relationen

$$V = \frac{R_{x_1} R_{x_2} - R(x_1 x_2)^2}{(x_1 - x_2)^2}$$

skrifva W under följande formen.

Om jag ponerar

$$s_1 = \frac{R(x_1 x_2) + \sqrt{R_{x_1}}\sqrt{R_{x_2}}}{2(x_1 - x_2)^2}$$

$$s_2 = \frac{R(x_1 x_2) - \sqrt{R_{x_1}}\sqrt{R_{x_2}}}{2(x_1 - x_2)^2}$$

Lecture notes by Sonya, in Swedish

and within a fortnight, it is said, she could converse sufficiently well at social gatherings. Soon she was able to follow the political debates in the press and after the first semester, she gave her lectures in Swedish.

Sonya was in Stockholm to work at the university. She had never taught before; she had not even attended a class as a student. In January 1884 she gave her first lecture in a course on partial differential equations. There were 12 registered students, but the auditorium was full; the others were students, professors, university officials, interested citizens who had come to be part of this historic occasion. She turned out to be an excellent lecturer. This was

fortunate since she did not receive a salary from the university but was forced to collect money from her students! Her probationary semester was a success and her students were enthusiastic.

In June 1884 Sonya was given a professorship at Stockholms högskola for a five-year period and she was overjoyed. She made inquiries about becoming a Swedish citizen but eventually gave up. In the end Stockholm could not compete with Berlin and Paris, where Sonya really felt at home and was well-known and accepted in mathematical circles.

Gösta Mittag - Leffler engaged Sonya to work for Acta Mathematica. She corresponded with many mathematicians around the world and developed a very strong network. Her numerous letters from the first years in Stockholm contain many interesting comments on actual mathematics research as well as a good dose of mathematical gossip.

Sonya took part in the social life of Stockholm. She was received by King Oscar II and subsequently wrote the following to Mittag-Leffler:

I have just visited the king, who was very nice. I had to give him a whole course on the theory of Helmholtz's Obertöne, he seemed very interested! (Cooke p. 108).

Sonya's first few years in Stockholm were very productive from a mathematics perspective. In 1886 she presented highly significant results on the solutions of the Euler equations for the rotation of a rigid body around a fixed point. One was a new special case where the equations could be completely integrated; the second was

proof that no other cases exist in which the solution is a meromorphic function of time. The new case represented physically a non-symmetric body, and the rotation of such a body under the influence of gravity is extremely complicated. The negative finding in the second result was equally important as it put an end to attempts to solve the equations using Abelian integrals. Sonya had thus in principle solved the problem, although there remained considerable work to be done to sort out the details. Sonya herself planned to work out explicitly how to express the parameters that describe the body's motion as functions of time.

Sonya was well aware that she must let others know about her results as soon as possible. To this end, she left for Paris. Sonya wrote to Mittag-Leffler:

M. Bertrand (the secretary of the French Academy) *evinces a very extraordinary benevolence towards me. Just imagine what he has thought up. Next Monday these gentlemen are to meet and propose the topic for the Grand Prix of the Academy for the year 1888. Bertrand has the idea of proposing as a topic precisely the problem on the rotation of a rigid body. That way I shall have the chance of obtaining the prize.* (Cooke p. 111).

In the end the topic was proposed, not for the Grand Prize, but for a less prestigious one, the Prix Bordin. The prize-money was the same though. Many other things occupied Sonya's mind in 1886-87 preventing her from getting to work in earnest on the problem for the Prix Bordin until just a few months before the deadline for submitting her paper. She struggled with transforming the integrals. She sought help from Weierstrass (who was too ill to respond), but in the end managed to solve the

SUR LE PROBLÈME DE LA ROTATION D'UN CORPS SOLIDE AUTOUR D'UN POINT FIXE[1]

PAR

SOPHIE KOWALEVSKI

à STOCKHOLM.

§ 1.

Le problème de la rotation d'un corps solide pesant autour d'un point fixe peut se ramener, comme on sait, à l'intégration du système d'équations différentielles suivant:

$$
\begin{aligned}
A\frac{dp}{dt} &= (B-C)qr + Mg(y_0\gamma'' - z_0\gamma'), & \frac{d\gamma}{dt} &= r\gamma' - q\gamma'',\\
B\frac{dq}{dt} &= (C-A)rp + Mg(z_0\gamma - x_0\gamma''), & \frac{d\gamma'}{dt} &= p\gamma'' - r\gamma, \qquad (1)\\
C\frac{dr}{dt} &= (A-B)pq + Mg(x_0\gamma' - y_0\gamma), & \frac{d\gamma''}{dt} &= q\gamma - p\gamma'.
\end{aligned}
$$

Les constantes A, B, C, Mg, x_0, y_0, z_0 qui figurent dans ces équations ont la signification suivante.

A, B, C sont les axes principaux de l'ellipsoïde d'inertie du corps considéré, relativement au point fixe.

M est la masse du corps;

g l'intensité de la force de gravité;

[1] Ce mémoire est le résumé d'un travail auquel l'Académie des Sciences de Paris, dans sa séance solennelle du 24 décembre 1888, a décerné le prix Bordin élevé de 3000 à 5000 francs.

Acta mathematica. 12. Imprimé le 22 janvier 1889.

First page of the Prix Bordin winning paper

problems on her own. She was able to submit a half-finished version by the deadline and asked for permission to send a revised version later on. Her request was granted. She submitted her revised version a couple of months later and was confident of its quality. Just before Christmas the prize-winner was announced: Mme Kovalevskaia. The French Academy was so impressed by the originality of the results that they doubled the prize-money. The committee report said:

..The author has not contented himself with adding a result of very high interest to those which were bequeathed us by Euler and Lagrange; he has made a profound study of the results due to him, in which all the resources of the modern theory of theta functions of two independent variables allow the complete solution to be given in the most precise and elegant form. One has thereby a new and memorable example of a problem of mechanics in which these transcendental functions figure, whose applications previously has been limited to pure analysis and geometry… (Cooke p. 115).

Sonya had reached a new goal. But it had taken its toll. She wrote to Mittag-Leffler a few weeks after receiving the prize:

.. I am receiving letters of congratulation from all over and by a strange derision of fate I have never in my life felt so unhappy as I feel at this moment. Unhappy as a dog; no, for the dogs' sake I hope that they cannot be as unhappy as men, and especially women, can be…. (Cooke p. 115).

The final years

Sonya was totally exhausted and close to a nervous breakdown. She had been working hard for several years. She brought her daughter Fufa to Stockholm in autumn 1886 and took her role as a mother seriously applying her own distinct ideas on a proper upbringing. Her sister Aniuta grew seriously ill the same year causing Sonya to be both distressed and eager to be of help and take care of her. Aniuta died in autumn 1887.

While Sonya was working on her paper for the Prix Bordin, she had classes to teach and additional duties attached to her position as a regular faculty member. She also had work to do for Acta Mathematica. She was torn between her professional and personal obligations, but met with no understanding from her colleagues. She asked for a leave of absence to visit Aniuta. The request was flatly denied and even looked upon as impertinence: male professors never asked for leave to take care of sick relatives. Her male colleagues looked for any sign of unprofessional behaviour. She once wrote a note as follows:

..Could you please tell me if you know anything about today's faculty meeting and whether you intend to go? I think that I myself must go so as not to give Peterson and Lecke the chance to say, "Professor Kovalevski naturally didn't come because she was occupied with the bazaar …" (Cook p. 112) (There was an upcoming bazaar at the Russian Embassy).

In spring 1887 Maksim Maksimovich Kovalevsky, a distant relative of Vladimir who Sonya had met in Paris some years earlier, arrived in Stockholm. He had been dismissed from his professorship in law at Moscow University due to his liberal ideas. Maksim was invited to lecture in many cities and knew many of the intellectuals of the day. This is also what brought him to Stockholm. Sonya and Maksim became close because, as Sonya explained, they were both homesick for Russia.

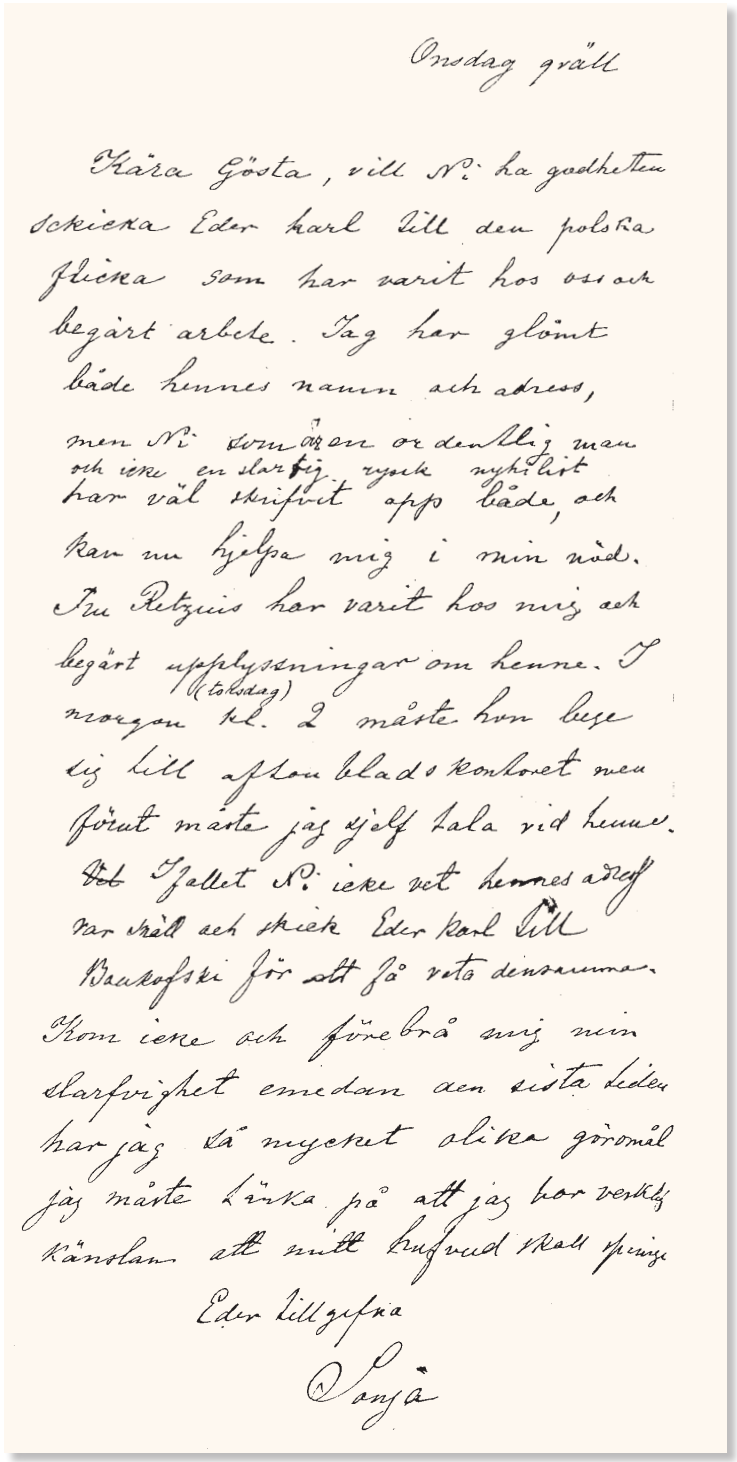
Sonya and her friend Ann-Charlotte Leffler

Sonya had become close friends with the sister of Gösta Mittag-Leffler, Ann-Charlotte Leffler, a well-known writer. Right from the start, Sonya with all of her experience, showed great interest in Ann-Charlotte's writing. The latter wrote in a biography of Sonya (Sonya Kovalevsky, 1895)

This was the commencement of the great influence she exercised on my writings afterwards. Her power of understanding and sympathising with the thoughts of others was so exceptional, her praise when she was pleased so warm and enthusiastic, her criticism so just, that for a receptive nature like mine, it was impossible to work without her approbation. (Koblitz p. 181):

In spring 1887, Sonya got an idea for a play in two parts contrasting a depressing picture of the lives of some young people with an inspiring picture of how their lives might have been. She convinced Ann-Charlotte Leffler to do the writing since her own Swedish was not good enough. The result was two plays: *The struggle for Happiness: How it was* and *How it might have been*, and it was staged once. Gösta Mittag-Leffler was very annoyed at this idea of Sonya, believing she was wasting her time and talent.

After winning the Prix Bordin, Sonya began to think about moving to Paris. Stockholm was becoming too provincial for her. She also entertained thoughts of a life with Maksim. To be appointed a position in Paris she would have to stay there



Onsdag gräll

Kära Gösta, vill Ni ha godheten
skicka Eder karl till den polska
flicka som har varit hos oss och
begärt arbete. Jag har glömt
både hennes namn och adress,
men Ni som äger en oerhörd minn
och icke en slarvig ryska nyhilist
har väl skrifvit upp både, och
kan nu hjelpa mig i min nöd.
Fru Retzius har varit hos mig och
begärt upplysningar om henne. I
morgon (tisdag) kl. 2 måste hon bese
sig till aftonbladskontoret men
förut måste jag själf tala vid henne.
Det fallet Ni icke vet hennes adress
var snäll och skicka Eder karl till
Baukofski för att få veta densamma.
Kom icke och förebrå mig min
slarfvighet emedan den sista tiden
har jag så mycket olösta göromål
jag måste tänka på att jag har verklig
känslan att mitt hufvud skall spring

Eder tillgifna

Sonja

Letter from Sonya to Gösta Mittag-Leffler

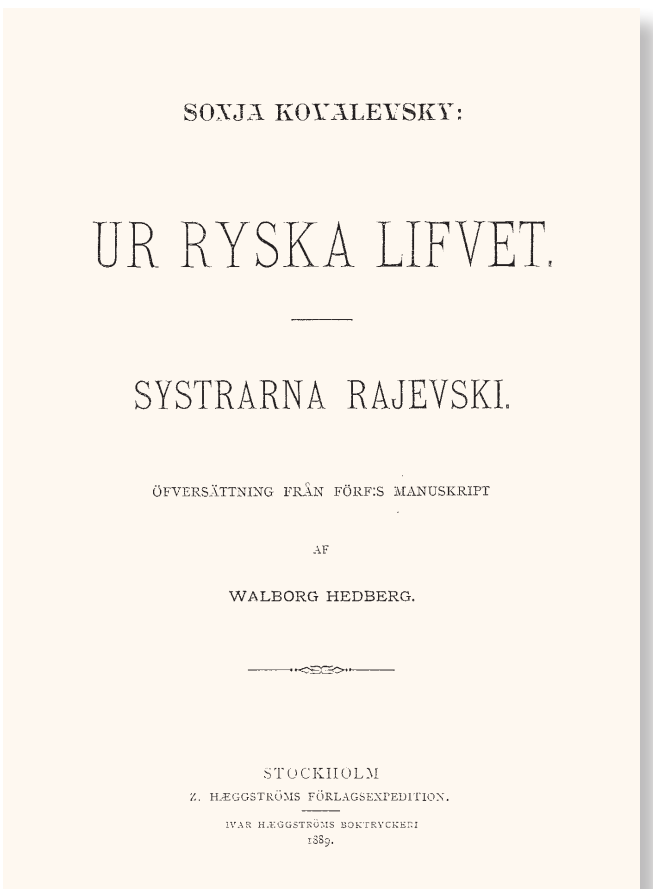
for at least six months and she would need a doctoral degree from a French university. She was granted a leave of absence from Stockholms högskola, but when Weierstrass heard about her plans he was furious, calling accepting a second degree in the same field the greatest insult to the faculty at Göttingen. Consequently, she eventually gave up her plan to leave Stockholm, and in May 1889 the university granted her tenure.

By autumn Sonya was back in Stockholm teaching and working on her mathematical problems. During her leave in 1889 she started writing down stories from her childhood. Her literary friends encouraged her to work seriously on them. Back in Sweden she started reading her memoirs aloud to a circle of women; Ellen Key was among them. Sonya read in Russian which was simultaneously interpreted into Swedish. They gave Sonya valuable advice, such as writing in the third person and changing names, and in late 1889 her book *Ur ryska lifvet. Systrarna Rajevski* appeared. *Memoirs of a Russian Childhood* was published in two issues of a Russian journal in summer 1890 and received enthusiastic reviews.

Sonya was working on another novel, The Nihilist Girl. Although unfinished at the time of her death, a Swedish and a Russian version were published in 1892, the latter printed in Geneva.The Nihilist Girl is the story of a young woman who sacrifices her personal life to marry a condemned revolutionary. It is based on a real event: a young Russian woman asked Sonya to find a way for her to help revolutionaries who had been arrested and together they concluded that marrying one of them

was the best way. The novel also contains autobiographical material from Sonya's return to Russia in 1874.

Sonya was appointed as a Correspondent Member of Russian Academy of Sciences and was nominated as a full member in 1890. A young mathematician, Markov, found a mistake in one of Sonya's elaborations of the Euler problem. Her claim was correct, but her proof contained a small gap. Markov's criticism and the growing opposition to allowing a woman to enter the Academy meant that she had no chance of being elected.



SONJA KOVALEVSKY:

UR RYSKA LIFVET.

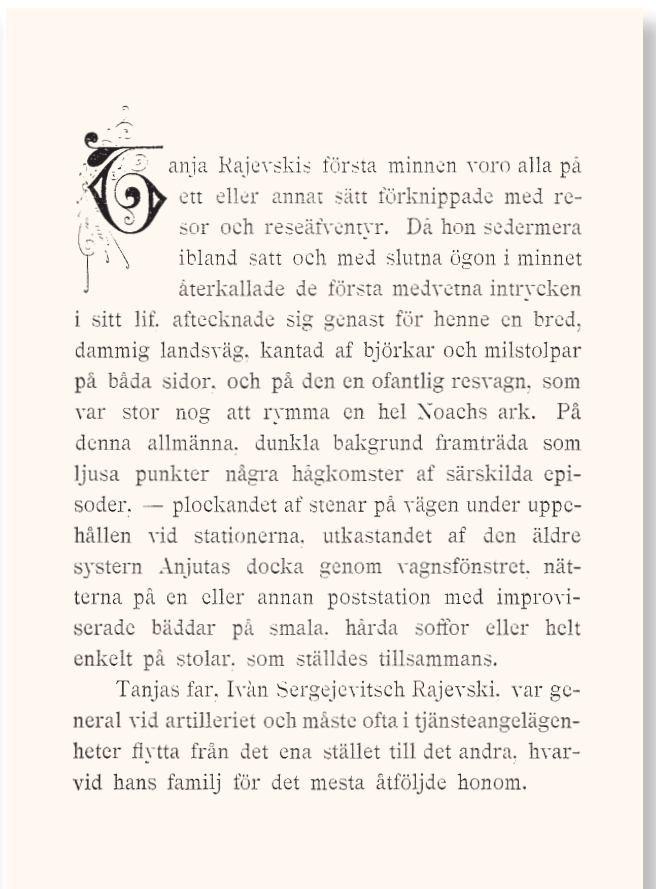
SYSTRARNA RAJEVSKI.

ÖFVERSÄTTNING FRÅN FÖRF:S MANUSKRIPT

AF

WALBORG HEDBERG.

STOCKHOLM
Z. HÆGGSTRÖMS FÖRLAGSEXPEDITION.
IVAR HÆGGSTRÖMS BOKTRYCKERI
1889.



Tanjas första minnen voro alla på ett eller annat sätt förknippade med resor och reseäfventyr. Då hon sedermera ibland satt och med slutna ögon i minnet återkallade de första medvetna intrycken i sitt lif, afteckade sig genast för henne en bred, dammig landsväg, kantad af björkar och milstolpar på båda sidor, och på den en ofantlig resvagn, som var stor nog att rymma en hel Noachs ark. På denna allmänna, dunkla bakgrund framträda som ljusa punkter några hågkomster af särskilda episoder, — plockandet af stenar på vägen under uppehållen vid stationerna, utkastandet af den äldre systern Anjutas docka genom vagnsfönstret, nätterna på en eller annan poststation med improviserade bäddar på smala, hårda soffor eller helt enkelt på stolar, som ställdes tillsammans.

Tanjas far, Iván Sergejevitsch Rajevski, var general vid artilleriet och måste ofta i tjänsteangelägenheter flytta från det ena stället till det andra, hvarvid hans familj för det mesta åtföljde honom.

First pages from Sonya's autobiographical notes The Rajevsky Sisters

She did not start on any new mathematics problems in 1998 and 1890, but instead worked on variations of the Euler problem. She also went back to one of her unpublished works from 1874. A paper was published after her death containing proof that the potential of a suitably shaped body is analytic at the boundary.

Sonya spent the 1890/1891 Christmas holidays with Maksim in France. She caught a cold and it became more severe during the long, cold and exhausting journey back to Stockholm. When she arrived she was seriously ill, bed-ridden and receiving constant care. For the first few days of her illness she was able to discuss mathematical problems with Mittag-Leffler and two writing projects with Ellen Key. She even dictated a final scene for one of them to Ellen Key. But after five days she fell into a coma and died on 10 February 1891, only 41 years of age.

News of Sonya's death shocked mathematicians and other intellectuals throughout Europe. A large crowd followed her funeral procession and bouquets and wreaths overflowed. She was buried at Norra Begravningsplatsen in Solna. Her grave is marked by a large Russian cross paid for by donations from her admirers in Russia, many of them women.

"Just emedan jag av den ensidige Strindberg blivit orättvist anfallen, gläder det mig att kunna visa den genialiske Strindberg min beundran. Vi kvinnor borde söka lära oss, vad männen ännu så litet kunna,att icke låta en persons svagheter eller misstag skymma bort hans verkliga betydelse."

Comment by Sonya on August Strindberg

Sonya as a mathematician

Those interested in gaining a closer insight into the mathematics of Sonya Kovalevskaia should refer to *The Mathematics of Sonya Kovalevskya* by Roger Cooke. Cooke writes:

The total corpus of her work presents us with a portrait of a competent, creative mathematician who produced some valuable work and a few works of minor importance, making an occasional mistake in the process. …she was a mathematician like most others, more gifted than most, but far more distracted from her work far more often as well. (Cooke p. 178)

Sonya is credited with having a more "applied" approach than, for instance, Weierstrass with his "pure" mathematics. It is true that several of her problems relate to physics, but the physical situation was mathematised well before Sonya worked on them. She herself had a pure interest in finding mathematical solutions to various sets of differential equations using new mathematical methods. The real significance of her award-winning work is, for instance, that she applied some very esoteric mathematics to physics problems.

No mathematician today cites her work directly, but this is true for most 19th century mathematics. Her work has been integrated into the foundation of present day mathematics. It is true, however, that Sonya worked on problems at the very heart of mathematics of her day, as is witnessed by the number of other well-reputed mathe-

maticians who worked on similar problems and in related areas. She corresponded with and met a great many mathematicians of her day and was a respected colleague.

The author shares the view of Roger Cooke when he writes: ...*she would not seem quite so remarkable if she lived a century later, when no one is surprised to see a competent woman mathematician. What one must remember is that this acceptance of what women can do is in no small part due to Kovalevskaia's achievements; and at least some of those achievements are grand on an absolute, not relative, scale.* (Cooke p. 179)

Bust of Sonya at the Institute Mittag-Leffler, by sculptor Walter Runeberg 1981.

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Sonya Kovalevsky and her life are the subject of the 1983 film *Berget på månens baksida* directed by Lennart Hjulström and a collection of stories by Alice Munro entitled *Too Much Happiness* from 2010.

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